2011-2012
IB MYP YEAR 4

SUMMATIVE ASSESSMENT

Year 9 Mathematics (Extended)

Name: Vivian Wu [9 Hope]

Date of task: 8th June, 2012 Time allowed: **1.5 hours (11:40 -13:10)**

Teacher: **Ms Li** / **Mr Millard** / **Mr So**

Student's **Performance** in Different **Criteria**			
A	3	C	3

Instructions

- Read the instructions for all questions carefully.
- All work must be hand written.
- All work, steps and proper units must be shown.
- A non-electronic dictionary is allowed.
- Use of calculator is allowed.

Advice:

- Read the criteria descriptors and task-specific rubrics carefully before you start your work. This will give you a clear understanding of what is required and what a high quality piece of work for this task must include. This way you give yourself the best chance of achieving the highest levels in this task.

- This assessment task will be assessed on Criterion **A & C**.
  - For Criteria **A**, the questions are all assigned with levels;
  - Criterion **C** will be assessed as an overall impression on the presentation of work in this assessment.

ASSESSMENT CRITERIA

Criterion A: KNOWLEDGE AND UNDERSTANDING

Achievement level	Task Specific Rubric	IBO Published Descriptor
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.
1–2 **Simple**	The student can solve <u>**some**</u> **simple** problems.	The student **generally** makes appropriate deductions when solving **simple** problems in **familiar** contexts.
3–4 **Complex**	The student can solve <u>**most**</u> **simple** and <u>**some**</u> **more complex** problems.	The student **generally** makes appropriate deductions when solving **more complex** problems in **familiar** contexts.
5–6 **Challenging**	The student can solve <u>**some**</u> **challenging** problem along with <u>**all**</u> different types of problems.	The student **generally** makes **appropriate** deductions when solving **challenging** problems in a **variety** of **familiar** contexts.
7–8 **Unfamiliar**	The student can solve <u>**most**</u> challenging and <u>**most**</u> **unfamiliar** problems along with <u>**all**</u> different types of problems.	The student **consistently** makes **appropriate** deductions when solving **challenging** problems in a **variety** of contexts including **unfamiliar** situations.

Criterion C: COMMUNICATION IN MATHEMATICS

Achievement level	Task Specific Rubric	IBO Published Descriptor
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.
1–2	The student should be able to explain <u>**some problems**</u> step by step. The lines of reasoning are <u>**difficult to follow**</u>.	The student shows **basic** use of mathematical language **and/or** forms of mathematical representation. The lines of reasoning are **difficult to follow**.
3–4	The student should be able to explain <u>**most problems**</u> step by step. The lines of reasoning are <u>**clear**</u> though <u>**not always**</u> **logical** or **complete**.	The student shows **sufficient** use of mathematical language **and** forms of mathematical representation. The lines of reasoning are **clear** though not always **logical** or **complete**. The student moves between different forms of representation **with some success**.
5–6	The student should be able to explain <u>**most problems**</u> step by step. The lines of reasoning are **concise**, **logical** and **complete**. The student use **correct unit** in the questions.	The student shows **good** use of mathematical language **and** forms of mathematical representation. The lines of reasoning are **concise**, **logical** and **complete**. The student moves **effectively** between different forms of representation.

A. SIMPLE PROBLEMS

Suggested time allocation for Question **1 to 5** is **15 minutes**.

1. Given the points A (–1 , 2) and B = (2 , *k*), find the value(s) of ***k*** such that the **length of line AB is 5 units**.

$\sqrt{[2-(-1)]^2+(k-2)^2}=5$

$\sqrt{3^2+k^2-4k+4}=5$

$\sqrt{k^2-4k+13}=5$

$k^2-4k+13=5,$

$k^2-4k=-8$

$-4k=\frac{-8}{k^2}$

$-k=\frac{-8}{k^2}\cdot\frac{1}{4}$

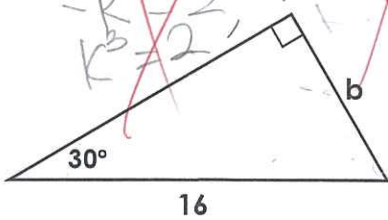
$-k=\frac{-2}{k^2}$

$\cancel{-k^3=-2,}$

$\cancel{k^3=2}$

$\cancel{k=1.26}$

2. In the figure on the right, find the value of **b** **without using calculator**.



$\sin 30^\circ=\frac{b}{16}$

$\frac{1}{2}=\frac{b}{16}$

$2b=16$

$b=8$

3. Given that the equation of the line **L1** is $y-2x=4$, which of the following line(s) is/are **parallel to L1**? Which of the following line(s) has/have **negative y-intercepts**?

L2: $y=-2x+4$ **L3:** $2y-4x-5=0$ **L4:** $-3y=2x+4$ **L5:** $6x-9=3y$

Explain your answers by showing your calculations.

$L1: y-2x=4$
$y=2x+4$

$L2: y=-2x+4$

$L3: 2y-4x-5=0$
$2y=4x+5$
$y=2x+\frac{5}{2}$

$L4: -3y=2x+4$
$y=\frac{-2x}{3}-\frac{4}{3}$

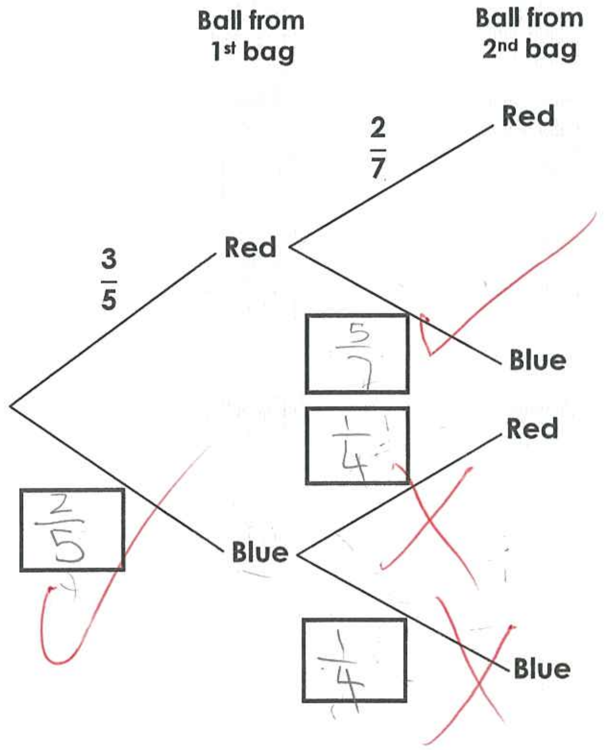
$L5: 6x-9=3y,\ y=2x-3$

$\because$ slope of L3, L4 and L5 are the same.

$\therefore$ L1 // L3, ~~L1 // L4~~, L1 // L5.

4. Loren has two bags. The **first** bag contains **3 red** balls and **2 blue** balls. The **second** bag contains **2 red** balls and **5 blue** balls. Loren takes **1 ball** at random from **each bag**.

(a) Complete the probability **tree diagram** by entering the **correct answers into the boxes.**



(b) Find the probability that Loren takes **two red balls**.

P(2 Red balls) = ____

5. Evaluate the following **without using calculator.**

$$\sin^2 23° + \cos^2 23° - \frac{\sin 45°}{\cos 45°}$$

$\sin^2 23° + \cos^2 23° - \frac{\sin 45°}{\cos 45°}$

$= 1 - \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$

$= 1 - \frac{\sqrt{2}}{2} \cdot \frac{2}{\sqrt{2}}$

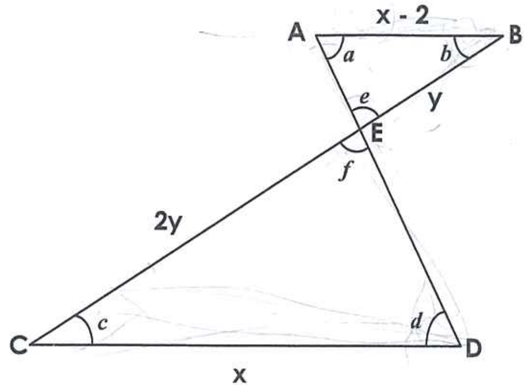
$= 1 - 1$

$= 0$

B. More complex problems

Suggested time allocation for Question **6 to 9** is **25 minutes**.

6. In the figure below, the line AB is parallel to the line CD and some dimensions are shown in terms of x or y.



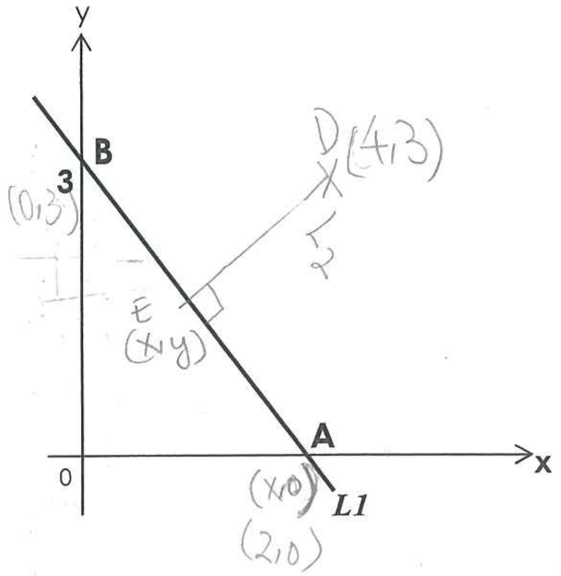
(a) Show that Δ **ABE** and Δ**DCE** are **similar**. State the reason(s) if necessary.

$\angle CED = \angle BEA$ (vert. opp. $\angle$s)
$\angle ABE = \angle DCE$ (alt. $\angle$s, AB//CD)
$\angle EAB = \angle EDC$ (alt. $\angle$s, AB//CD)
$\therefore \triangle ABE \sim \triangle DCE$ (equiangular)

(b) Find the value of **x**.

$\because \triangle ABE \sim \triangle DCE$ (proved)
$\therefore \frac{BE}{CE} = \frac{AB}{DC}$ (corr. sides, $\sim \triangle$s)
$\frac{y}{2y} = \frac{x-2}{x}$
$xy = 2yx - 4y$
$x = \frac{2yx - 4y}{y}$
$x = \frac{2y(x-2)}{y}$, $x = 2x - 4$, $x = 4$

7. In the graph on the right, a line $L1$ cuts the x-axis and y-axis at point **A** and **B** respectively. The y-intercept is 3.



(a) If the area of the triangle AOB is 3 square units, find the **equation** of $L1$. Express your answer in **slope-intercept form**.

$BO = \sqrt{(0-0)^2+(3-0)^2}$
$= \sqrt{0+9}$
$= \sqrt{9}$
$= 3$

Let (x,0) be the coordinates of A.

$\because \frac{AO \cdot BO}{2} = 3$

$\therefore AO \cdot 3 = 6$

$AO = 6 \times \frac{1}{3}$

$AO = 2$ units

$\sqrt{(x-0)^2+(0-0)^2} = 2, \sqrt{x^2-0+0+0^2} = 2, x^2 = 4, x = 2$

$mAB = \frac{3-0}{0-2} = -\frac{3}{2}$

$\therefore$ Equation of L_1 is $y = -\frac{3}{2}x + 3$

(b) If a line $L2$ is **perpendicular** to $L1$ and two lines intersect at point **D(4,-3)**, find the equation of $L2$. Express your answer in **general form**.

$y = mx + c$

$ax + by + c = 0$

Let (x,y) be the coordinates of E.

$mL_1 \times mL_2 = -1$

$\frac{y+3}{x-4} \cdot \frac{3-0}{0-2} = -1$

$\frac{y-3}{x-4} \cdot -\frac{3}{2} = -1$

$\frac{-3y+9}{2x-8} = -1$

$-3y+9 = -2x+8$

$3y+8 = 2x+9$

$3y-2x-1=0$

$\therefore$ The equation of L_2 is $3y-2x-1=0$

8. In a certain dice game, the player throws **two** typical unbiased **six-faces dice** and receives **$5** if the sum is **7 or 11**, otherwise he or she **pays $2**.

(a) Calculate the probability of obtaining the **sum of 7 or 11** when you **throw the two dice once**.

1\2	1	2	3	4	5	6
1	/	1,2	1,3	1,4	1,5	1,6
2	2,1	/	2,3	2,4	2,5	2,6
3	3,1	3,2	/	3,4	3,5	3,6
4	4,1	4,2	4,3	/	4,5	4,6
5	5,1	5,2	5,3	5,4	/	5,6
6	6,1	6,2	6,3	6,4	6,5	/

$P(\text{sum of 7 or 11}) = \frac{8}{30} = \frac{4}{15}$

(b) If you play the game **18 times**, calculate the **amount of money** you **expect to gain or lose**.

$\text{Gain} = \frac{4}{15} \times 18 \times 5 = \$24$

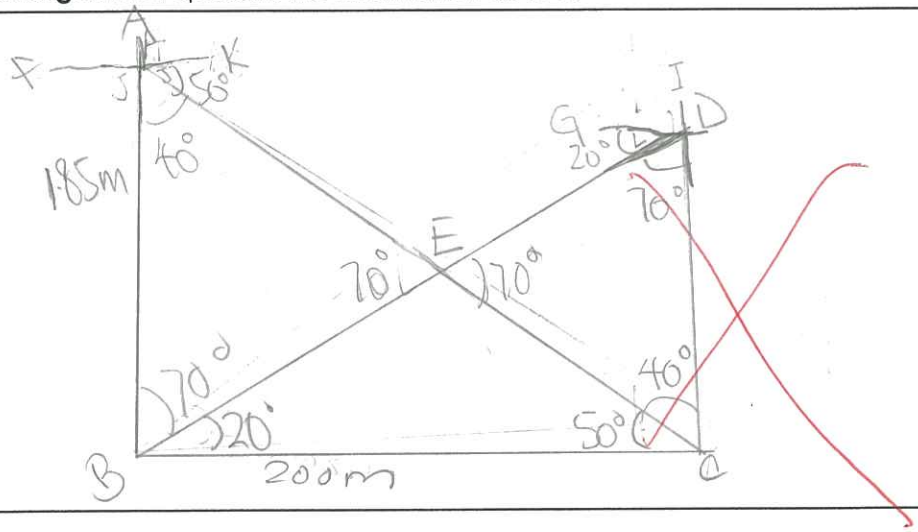
I expect to gain $24.

$\text{Lose} = \frac{11}{15} \times 18 \times 2 = \$26.4$

I expect to lose $26.4

9. Mr Bolivar, a volunteer fireman who is 1.85 m tall, is running towards a burning building where there is a fire on the roof. Initially, his angle of elevation to the roof is 20°. He runs for 200 m and now his angle of elevation is 50°. Assume that <u>the ground is horizontal and the building is vertical</u>.

(a) Sketch a **diagram** to represent the information above.



(b) How tall is the building? Correct your answer to the **nearest meter**.

AF // BC (by construction)
DG // BC (by construction)
$\therefore \angle KJE = 50^\circ$ (given)
$\therefore \angle ECB = 50^\circ$ (alt ∠s, AF//BC)
$\therefore \angle GLE = 20^\circ$
$\therefore \angle EBC = 20^\circ$ (alt ∠s, AF//BC)

$\tan 20^\circ = \frac{DC}{BC}$

$\tan 20^\circ = \frac{DC}{200}$

$DC = \tan 20^\circ \times 200$

$DC = 73m$

The building is 73m tall.

C. CHALLENGING PROBLEM

Suggested time allocation for Question **10 and 11** is **30 minutes**.

10. Ship **A** leaves the harbor H on a bearing **150°** with a speed of **40** km/hr. At the same time, Ship **B** leaves harbor H on a bearing **210°** with a speed of **40** km/hr.

(a) **After 12 minutes**, how far did ship A and ship B travel?

Ship A $= 40 \times \frac{12}{60}$
$= \cancel{40}^{8} \times \frac{1}{\cancel{5}}$
$= 8$

Ship B $= 40 \times \frac{12}{60}$
$= \cancel{40}^{8} \times \frac{1}{\cancel{5}}$
$= 8$

Ship A and ship B travels 8km after 12 minutes.

(b) Sketch a **diagram** to represent the information above **12 minutes after the two ships left the harbor.**

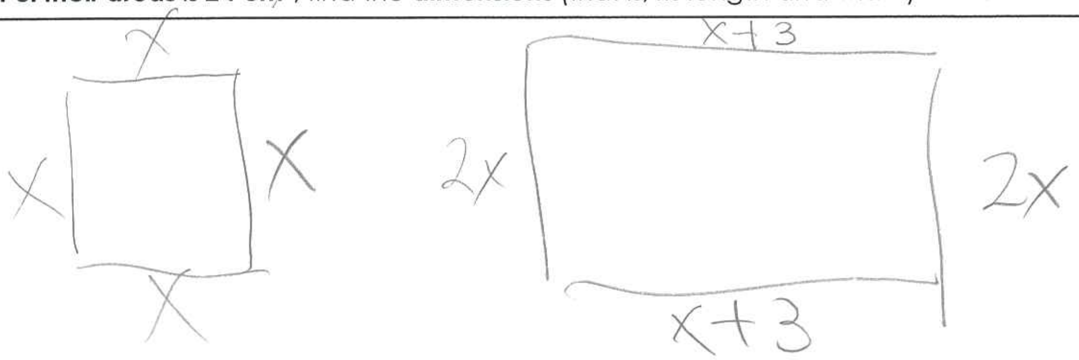
(c) Find the **true bearing from Ship A to Ship B** 12 minutes after they left the harbor.

(d) Find the **distance between the two ships** 12 minutes after they left the harbor. Give your answer to the **nearest meter.**

11. The properties of a rectangle and a square are given below:

- The length of the rectangle is 3 cm longer than the side of the square.
- The width of the rectangle is double the length of the side of the square.

If the **sum of their areas** is **24 cm²**, find the **dimensions** (that is, its length and width) of the rectangle.



Let x be the length of the square.
Let 2x be the width of the rectangle.
Let x+3 be the length of the rectangle.

D. Unfamiliar problems (Suggested time allocation for <u>Question **12 and 13** is **30 minutes**</u>.)

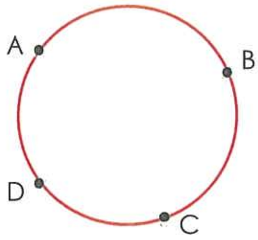
12. At noon, Tom and Pete both park at the same starting point. Tom starts to ride his bike at 8 miles/hr. Two hours later, Pete starts after Tom on a bicycle at 12 miles/hr.

(a) How far will Tom have ridden before he is **overtaken by Pete**?

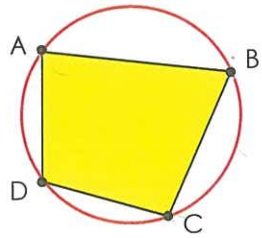
(b) At what time will Tom and Pete be **8 miles** apart?

13. Please read the following information and then do the proof on next page.

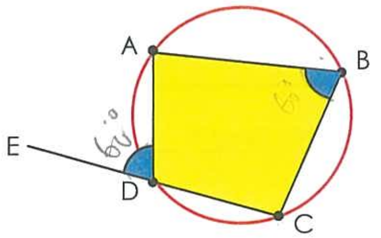
Points lie on the **same circle**, as the diagram below, are said to be **concyclic**. For example, A, B, C and D are **concyclic points**.



If the vertices of a **quadrilateral** lie on a **circle**, as the diagram below, then the quadrilateral is said to be **cyclic**. For example, ABCD is a **cyclic quadrilateral** since the vertices A, B, C and D lie on the circle.

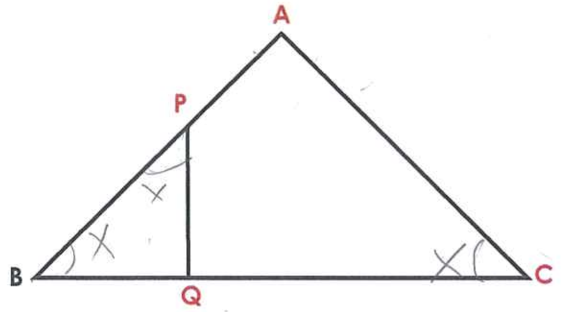


If the side CD is produced (i.e. extended) to E, as the diagram below, then $\angle$**ADE** is called the **exterior angle of the cyclic quadrilateral** ABCD, and $\angle$**ABC** is said to be the **interior opposite angle**.



Theorem: **If $\angle ADE = \angle ABC$, then A, B, C and D are concyclic. (ext.$\angle$, int. opp.$\angle$)**

In the figure below, **ΔABC** and **ΔBPQ** are **isosceles** triangles such that **AB = AC** and **BQ = PQ**.
Using the provided information about the concyclic points and cyclic quadrilateral, **prove** that **A, P, Q and C are concyclic**.



End of Assessment